Unit 3 | Curve Sketching

I. Sketching Polynomial Functions

Prior: Sketch $f(x) = x^3 + 2x^2 - 25x - 50$

1. Find the roots
2. End Behavior
3. In between behavior
4. Domain
5. Sketch
6. Max/Min
7. Range
8. Intervals of Inc/Dec

(+) even : $\uparrow \dots \uparrow$
(−) even : $\downarrow \dots \downarrow$
(+) odd : $\downarrow \dots \uparrow$
(−) odd : $\uparrow \dots \downarrow$

Calculus to the rescue!

A. Increasing / Decreasing Test

1. If $f'(x) > 0$ on an interval, then $f(x)$ is increasing on that interval.
2. If $f'(x) < 0$ on an interval, then $f(x)$ is decreasing on that interval.

Ex 1 where is $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ increasing & decreasing

approach: $f'(x) > 0$ and $f'(x) < 0$

$$f'(x) = 12x^3 - 12x^2 - 24x = 0$$

$$12x(x-2)(x+1) = 0$$

CPs $x = 0$, $x = 2$, $x = -1$

Int	$f'(x)$	conclusion
$(-\infty, -1)$	−	$f(x)$ is dec.
$(-1, 0)$	+	$f(x)$ is inc.
$(0, 2)$	−	$f(x)$ is dec.
$(2, \infty)$	+	$f(x)$ is inc.

∴ Decreasing: $(-\infty, -1) \cup (0, 2)$

Increasing: $(-1, 0) \cup (2, \infty)$

Note: Definition: A critical number/point of a function, $f(x)$, is a number c in the domain of $f(x)$ such that either $f'(c)=0$ or $f'(c)=$ does not exist

Slope is 0

derivative DNE

B. The First Derivative Test

Suppose that c is a critical point of a continuous function f.

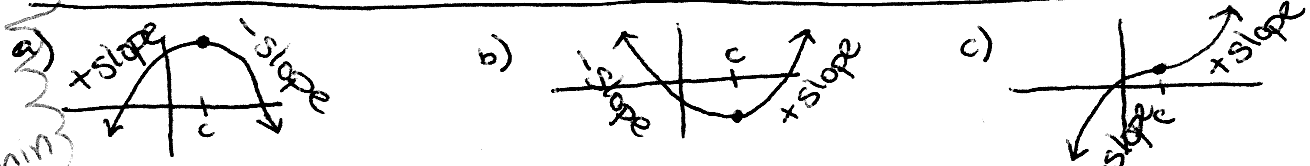
a) If f' changes from positive to negative at c, then f has a relative maximum at c.

b) If f' changes from negative to positive at c, then f has a relative minimum at c.

c) If f' does not change signs at c, then f has no relative maximum or minimum at c

relative = local

the absolute max/min is the highest/lowest max or min.



local max   local min   no max/min

global = absolute

$\boxed{\text{ex 1}}$ Where does $f(x)=x^2+3x-4$ have a minimum?

Approach: $f'(x)$ changes from $-$ to $+$

$f'(x)=2x+3$

cp $x=-\frac{3}{2}$

int	$f'(x)$	conclusion
$(-\infty,-\frac{3}{2})$	$-$	$f(x)$ is dec.
$(-\frac{3}{2},\infty)$	$+$	$f(x)$ is inc.

$\leftarrow \therefore$ min

A.P. answer: $f(x)$ has a min at $x=-\frac{3}{2}$ because $f'(x)$ changes from negative to positive.

AP level example

$\boxed{\text{ex2}}$ If $f(x) = \frac{1}{3}x^3 - 4x^2 + 12x - 5$ and the domain is the set of all real numbers x such that $\underline{0 \le x \le 9}$, then the absolute maximum value of the function $f(x)$ occurs when $x = ?$

Approach: find maximums, which is highest?
max: $f'(x)$ changes from + to −

$f'(x) = x^2 - 8x + 12$

$(x-6)(x-2) = 0$

c.p. $x = 6$ $x = 2$

and endpoints:

$x = 0$ $x = 9$

int	f(x)	f'(x)	conclusion
0	$(0,-5)$	+	inc. ⟵ ∴ min @ $x=0$
$(0,2)$	╳	+	inc.
2	$(2,\frac{17}{3})$	0	at rest ⟵ ∴ max @ $x=2$
$(2,6)$	╳	−	dec.
6	$(6,-5)$	0	at rest ⟵ ∴ min @ $x=6$
$(6,9)$	╳	+	Inc.
9	$(9,22)$	+	inc. ⟵ ∴ max @ $x=9$

include to find the y-values

helpful for graphing

$\boxed{\text{absolute max value at } x=9 \text{ is } 22.}$

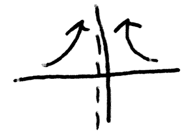
II Graphing Rational, Radical, Trig & Log Functions

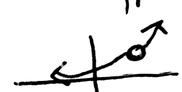
It's the same process as polynomials, except for domain issues; they will be additional critical points or possible inflection points, and horizontal and vertical asymptotes.

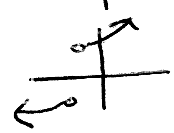
> Inflection points of a function f occur at c such that $f''(c)=0$ <u>and</u> $f''(x)$ changes from positive to negative or vice versa at c.

Domain issues; they will be critical points or possible inflection points

when $f(x) = DNE$

* asymptote  $\frac{x}{x-2} \Rightarrow$ V.A. @ $x=2$

* hole  $\frac{x\cancel{(x-2)}}{\cancel{(x-2)}} \Rightarrow$ hole @ $x=2$

* jump  piecewise

$\boxed{ex1}$ Sketch the graph of $f(x) = \frac{2x^2}{x^2-1}$ on the interval $[-5,4]$.

Approach: rational $\Rightarrow$ domain issues
f' for c.p. & f'' for ip

Domain issues: $x^2-1=0$
$x = \pm 1$

Boundary: $x=-5$
$x=4$

$$f'(x) = \frac{(x^2-1)(4x)-(2x^2)(2x)}{(x^2-1)^2} = \frac{\cancel{4x^3}-4x-\cancel{4x^3}}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2}$$

$x=0$

$x=\pm 1$ ← $f'(x) = DNE$

$$f''(x) = \frac{4(3x^2+1)}{(x^2-1)^3}$$

$\cancel{x = \sqrt{-\tfrac{1}{3}}}$

$x = \pm 1$ ← $f''(x) = DNE$

int	$f(x)$	$f'(x)$	$f''(x)$	conclusion about $f(x)$
-5	$(-5, \frac{25}{12})$	+	+	inc, CU
$(-5,-1)$		+	+	inc, CU
-1	DNE	DNE	DNE	?
$(-1,0)$		+	−	inc, CD
0	$(0,0)$	0	−	at rest, CD ∴ max
$(0,1)$		−	−	dec, CD
1	DNE	DNE	DNE	?
$(1,4)$		−	+	dec, CU
4	$(4, \frac{32}{15})$	−	+	dec, CU

DNEs may be VAs

$\lim_{x \to -1} \frac{2x^2}{x^2-1} = \frac{2}{0} = \infty$

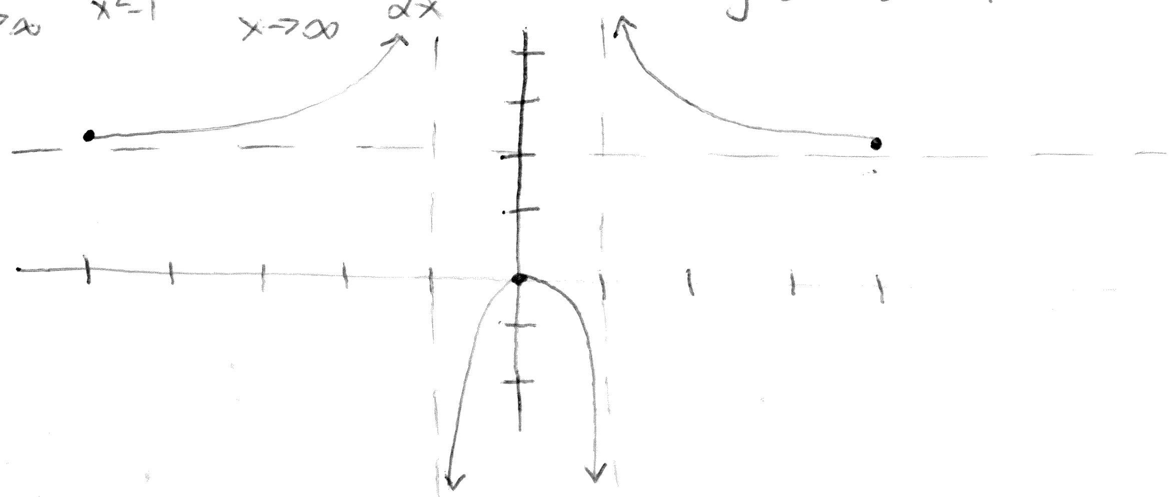
∴ $x=-1$ is a VA

$\lim_{x \to 1} \frac{2x^2}{x^2-1} = \frac{2}{0} = \infty$

∴ $x=1$ is a VA

Check for HA or OA:

$\lim_{x \to \infty} \frac{2x^2}{x^2-1} = \lim_{x \to \infty} \frac{4x}{2x} = 2$ ∴ $y=2$ is HA



$\boxed{\text{ex2}}$ Sketch $f(x) = \sin x + \cos x$, $0 \leq x \leq \pi$

$f'(x) = \cos x - \sin x = 0$

$\cos x = \sin x$

c.p. $x = \frac{\pi}{4}$

$f''(x) = -\sin x - \cos x = 0$

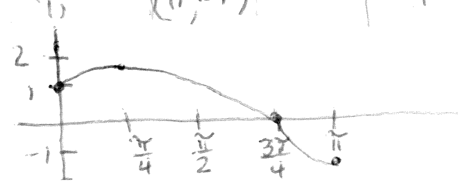
$-\sin x = \cos x$

p.i.p $x = \frac{3\pi}{4}$

boundary: $x=0$

$x=\pi$

int	$f(x)$	$f'(x)$	$f''(x)$	conclusion about $f(x)$
0	$(0,1)$	+	−	inc, CD
$(0,\frac{\pi}{4})$		+	−	inc, CD
$\frac{\pi}{4}$	$(\frac{\pi}{4}, \sqrt{2})$	0	−	at rest, CD ∴ max
$(\frac{\pi}{4}, \frac{3\pi}{4})$		−	−	dec, CD
$\frac{3\pi}{4}$	$(\frac{3\pi}{4}, 0)$	−	0	dec ∴ IP
$(\frac{3\pi}{4}, \pi)$		−	+	dec, CU
π	$(\pi,-1)$	−	+	dec, CU



III. Conceptual Curve Sketching (3C)

A. Facts:

$f'(a)=0$	a is a critical point of f(x)
$f'(a)=DNE$	a is a critical point of f(x)
$f'(a)<0$	f(x) is decreasing
$f'(a)>0$	f(x) is increasing
$f''(a)=0$	a is a possible inflection point
$f''(a)=DNE$	a is a possible inflection point
$f''(a)<0$	f(x) is concave down
$f''(a)>0$	f(x) is concave up
$f(a)=DNE$	f(x) has a hole, jump or asymptote
$f(a)>0$	f(x) is <u>above</u> the x-axis
$f(a)<0$	f(x) is <u>below</u> the x-axis

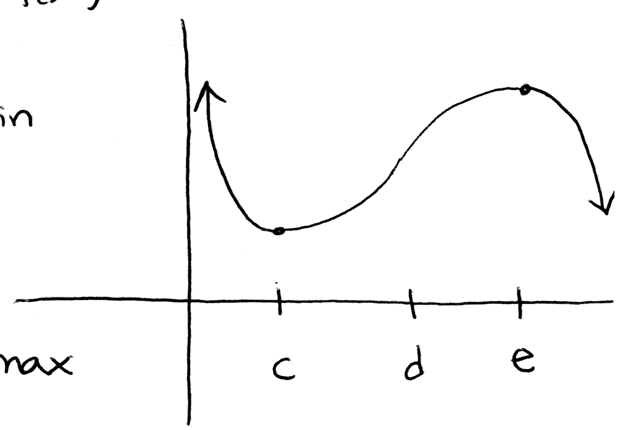
$\boxed{ex1}$ Sketch a graph that satisfies the following:

1. $c<d<e$ 2. $f'(c)=0$ 3. $f'(d)=1$ 4. $f''(d)=0$

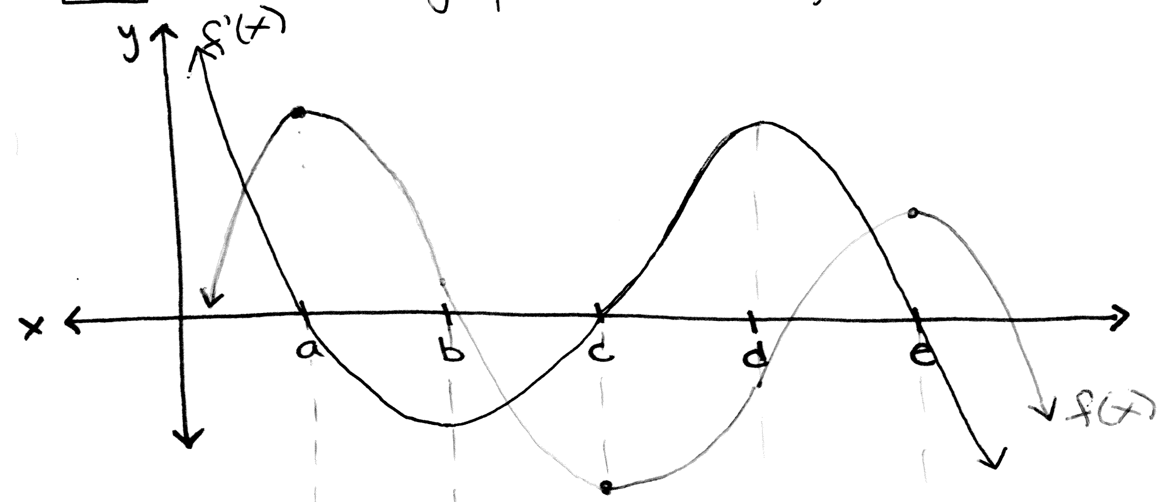
5. $f'(e)=0$ 6. $f''(x)>0$ if $x<d$

7. $f''(x)<0$ if $x>d$

int	f(x)	f'(x)	f''(x)	conclusion about f(x)
$(-\infty, c)$			+	CU
c		0	+	at rest, CU ∴ min
(c, d)			+	CU
d		+	0	inc, ∴ IP
(d, e)			−	CD
e		0	−	at rest, CD ∴ max
(e, ∞)			−	CD



$\boxed{\text{ex2}}$ Given the graph of $f'(x)$, sketch $f(x)$.



⊙ Set up where CPs & IPs occur
* $f'(x)=0$
* $f''(x)=0$

		a		b		c		d		e	
$f'(x)$	+	0	−	−	−	0	+	+	+	0	−
$f''(x)$	−	−	−	0	+	+	+	0	−	−	−
conclusion about f(x)	inc CD	max	dec CD	IP	dec CU	min	inc CU	IP	inc CD	max	dec CD

looking at the value of the function shown

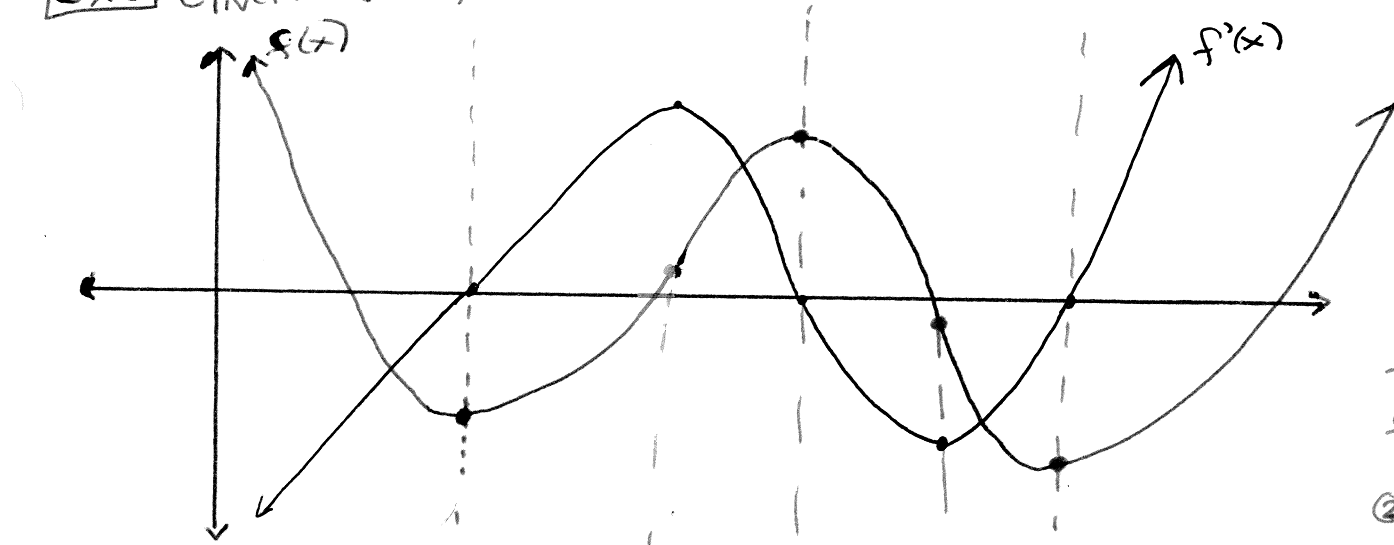
looking at the slopes of the function shown

slopes of f' represent f''

Note: There will always be more than 1 answer to this type of problem b/c no specific points are given for f(x).

Note: If you're given the graph of $f'(x)$, maximums & minimums occur when $f'(x)$ crosses the x-axis.

Ex 3 Given $f(x)$, sketch $f'(x)$.



① Setup C.Ps & IPs
$f'(x)=0$
$f''(x)=0$

② Fill out $f'(x)$ {slopes}

③ Fill out $f''(x)$

④ Make a conclusion

$f'(x)$: $-$ 0 $+$ $+$ $+$ 0 $-$ $-$ $-$ 0 $+$

$f''(x)$: $+$ $+$ $+$ 0 $-$ $-$ $-$ 0 $+$ $+$ $+$

Make a conclusion about $f'(x)$:

- * below the x-axis * inc
- on/through x-axis
- * above the x-axis * inc
- max
- * above the x-axis * dec
- on/through x-axis
- * below the x-axis * dec
- min
- * below * inc
- on/through x-axis
- * above * inc

Note: When $f(x)$ has a max or min, $f'(x)$ will cross through the x-axis

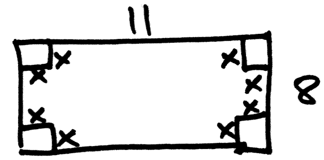
Note: when $f(x)$ has an inflection point, $f'(x)$ will have a max or min.

IV Applications of the Derivative

A. Optimization (3D)

$\boxed{\text{ex1}}$ From an 8ft by 11ft size rectangle, what size square do I cut off of the corners to maximize Volume?

Pre-Cal



$V = l \times w \times h$

$V = (11-2x)(8-2x)x$

$V = 4x^3 - 38x^2 + 88x$

maximize but $x = \frac{-b}{2a}$ only works for quadratics ☹

Calculus to the Rescue!

① Maximum occurs where f'(x) changes from positive to negative {first derivative test}

② Maximum occurs at a critical point where $f''(x) < 0$ aka CD {second derivative test}

$V' = 12x^2 - 76x + 88 = 0$ c.p. $x = 1.525$ or $\cancel{x = 4.808}$

check to make sure!

① 1st deriv. test.

int	$f'(x)$
$(0, 1.525)$	+
$(1.525, 4)$	−

$\therefore$ max at $x = 1.525$

$\therefore$ The slopes change from positive to negative at 1.525, so that is where the maximim Volume occurs

② 2nd deriv. test

$V'' = 24x - 76$

$V''(1.525) = 24(1.525) - 76 = -$ $\therefore$ CD

$\therefore$ The function is concave down at the critical point of 1.525, so that is where the max Vol occurs!

$\boxed{\text{ex 2}}$ What is the maximum acceleration on the interval $0 \le t \le 3$ by the particle whose velocity is given by $V(t) = t^3 - 3t^2 + 12t + 4$?

$a(t) = 3t^2 - 6t + 12 \leftarrow$ maximize

$a'(t) = 6t - 6 = 0$

$t = 1$ c.p.

and

$t=0$ $t=3$ b.p.

int	$a'(t)$
$(0,1)$	$-$
$(1,3)$	$+$

$\therefore$ min

so check the boundary points!

$a(0) = 12$

$a(3) = 21$

$\boxed{\therefore \text{ the max acceleration is } 21 \text{ units/time}^2}$

$\boxed{\text{ex 3}}$ Find the shortest distance from $\overset{\text{point 2}}{P}(2,0)$ to the curve $y^2 - x^2 = 1$.

distance formula:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\rightarrow y^2 = x^2 + 1 \quad \therefore y = \pm\sqrt{x^2+1}$

point 1

$(x, \pm\sqrt{x^2+1})$

$d = \sqrt{(x-2)^2 + (\pm\sqrt{x^2+1})^2}$

$d = \sqrt{x^2 - 4x + 4 + x^2 + 1}$

$d = \sqrt{2x^2 - 4x + 5} \leftarrow$ minimize

$d' = \frac{1}{2}(2x^2 - 4x + 5)^{-1/2}(4x - 4)$

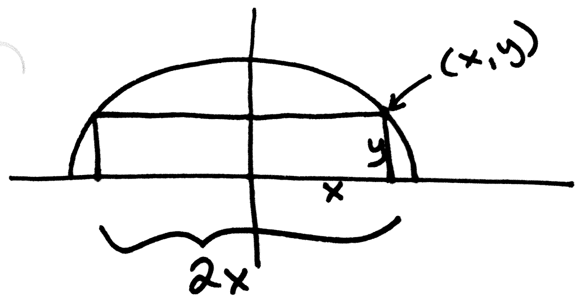
$d' = \dfrac{2x-2}{\sqrt{2x^2-4x+5}} = 0 \quad \therefore 2x - 2 = 0$

$x = 1$ c.p.

$d(1) = \sqrt{2(1)^2 - 4(1) + 5} = \boxed{\sqrt{3} \text{ units}}$

$\boxed{\text{ex4}}$ Find the area of the largest rectangle that can be inscribed in a semi circle of radius r.

everything is going to be in terms of "r"



$A = 2xy$

circle:

$x^2 + y^2 = r^2$

$y = \sqrt{r^2 - x^2}$

$A = 2x(r^2 - x^2)^{1/2}$ maximize

$$A' = 2x \cdot \frac{1}{2}(r^2 - x^2)^{-1/2}(-2x) + (r^2 - x^2)^{1/2} \cdot (2)$$

$$A' = \frac{-2x^2}{\sqrt{r^2 - x^2}} + 2\sqrt{r^2 - x^2} = 0$$

$$= -2x^2 + 2(r^2 - x^2) = 0$$

$$-2x^2 + 2r^2 - 2x^2 = 0$$

$$-4x^2 = -2r^2$$

$$x^2 = \frac{r^2}{2}$$

$$x = \pm\sqrt{\frac{r^2}{2}}$$

$$x = \pm\frac{r}{\sqrt{2}}$$

int	$A'(x)$	
$(-\infty, -\frac{r}{\sqrt{2}})$	$-$	
$(-\frac{r}{\sqrt{2}}, \frac{r}{\sqrt{2}})$	$+$	$\therefore$ min
$(\frac{r}{\sqrt{2}}, \infty)$	$-$	$\therefore$ max

so $x = \frac{r}{\sqrt{2}}$ gives the max area

$$A\left(\frac{r}{\sqrt{2}}\right) = 2 \cdot \frac{r}{\sqrt{2}}\left(r^2 - \left(\frac{r}{\sqrt{2}}\right)^2\right)^{1/2}$$

$$= \frac{2r}{\sqrt{2}}\left(r^2 - \frac{r^2}{2}\right)^{1/2}$$

$$= \frac{2r}{\sqrt{2}}\left(\frac{2r^2 - r^2}{2}\right)^{1/2}$$

$$= \frac{2r}{\sqrt{2}}\left(\frac{r^2}{2}\right)^{1/2}$$

$$\frac{2r}{\sqrt{2}} \cdot \frac{r}{\sqrt{2}} = \frac{2r^2}{2} = r^2$$

$\therefore$ the max area is

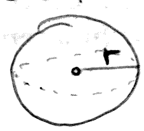
$A = r^2$

V Related Rates (3E)

A. Rates of change will be dependent on time.
we will use the derivative to find the specified rates of change {slope}.

$\boxed{\text{ex 1}}$ Air is being pumped into a spherical balloon at a rate of $20\ in^3/min$. At what rate is the radius changing when the radius is 3 in long?

Approach:



adding <u>Volume</u> at a rate of $20\ in^3/min$

$\hookrightarrow \frac{dV}{dt} = 20$

change in Volume with respect to time

want: the rate of change of the radius: $\rightarrow \frac{dr}{dt}$

$V = \frac{4}{3}\pi r^3$ ← r & V are <u>both</u> changing depending on the time ∴ implicit differentiation is used with respect to time

$$\frac{dV}{dt} = \cancel{3}\frac{4}{\cancel{3}}\pi r^2 \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

we want to find the rate of change in the radius, $\frac{dr}{dt}$.

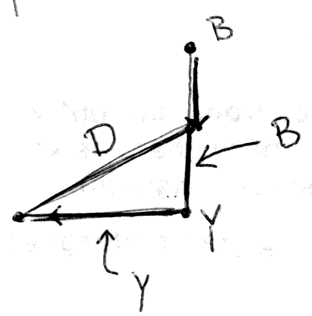
given: $\frac{dV}{dt} = 20$, $r = 3$

$$20 = 4\pi (3)^2 \frac{dr}{dt}$$

$$\boxed{\frac{dr}{dt} = \frac{20}{36\pi}}$$

$\boxed{\text{ex2}}$ At 8 pm, a boat is 20 km due North of a yacht. The boat travels South at a rate of 9 km/hr and the yacht travels due West at 12 km/hr. At 9:20 pm at what rate is the distance between the two changing?

Approach: Draw!



distance:
- ★ $d = rt$
- ★ pythagorean theorem
- ★ 1 hr 20 min $= \frac{4}{3}$ hr

All 3 change with respect to time

$$B^2 + Y^2 = D^2$$

$$2B \cdot \frac{dB}{dt} + 2Y \cdot \frac{dY}{dt} = 2D \cdot \frac{dD}{dt}$$

want: $\frac{dD}{dt}$

given: $\frac{dB}{dt} = 9$

$\frac{dY}{dt} = 12$

$$2(8)(9) + 2(16)(12) = 2(8\sqrt{5})\frac{dD}{dt}$$

$$\frac{528}{2(8\sqrt{5})} = \frac{dD}{dt}$$

$$\boxed{\frac{dD}{dt} = 73.79 \text{ Km/hr}}$$

$d = rt$

$B = 20 - 9\left(\frac{4}{3}\right) = 8$

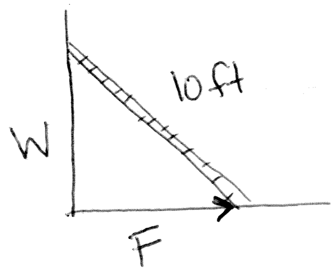
$Y = 12 \cdot \frac{4}{3} = 16$

$B^2 + Y^2 = D^2$

$8^2 + 16^2 = D^2$

$D = 8\sqrt{5}$

$\boxed{\text{ex3}}$ A ladder that is 10 ft long rests against a wall. If the bottom of the ladder is sliding away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom reaches 6 ft away?



$$W^2 + F^2 = 10^2$$

$$2W\frac{dW}{dt} + 2F\frac{dF}{dt} = 0$$

want: $\frac{dW}{dt}$

given: $\frac{dF}{dt} = 1$

$F = 6$

$W^2 + 6^2 = 10^2$

$W = 8$

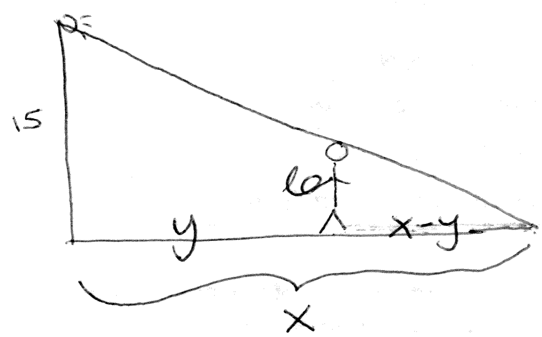
$$2(8)\frac{dW}{dt} + 2(6)(1) = 0$$

$$\boxed{\frac{dW}{dt} = -\frac{3}{4} \text{ ft/s}}$$

negative because it's sliding <u>down</u>

$\boxed{\text{ex 4}}$ A woman that is 6 ft tall walks away from a light post that is 15 ft tall at a rate of 5 ft/sec. At what rate is the tip of her shadow moving? How fast is the length of her shadow changing?

① Draw



given: $\frac{dy}{dt} = 5$

want: $\frac{dx}{dt}$

② Relate the sides/set up proportions

$$\frac{15}{x} = \frac{6}{x-y}$$

$$15(x-y) = 6x$$

$$15x - 15y = 6x$$

$$9x - 15y = 0$$

③ Implicit differentiation

$$9\frac{dx}{dt} - 15\frac{dy}{dt} = 0$$

④ Plug in

$$9\frac{dx}{dt} - 15(5) = 0$$

$$\frac{dx}{dt} = \frac{75}{9}$$

$$\boxed{\frac{dx}{dt} = \frac{25}{3} \text{ ft/sec}}$$

units!

treat (x-y) as <u>one</u> variable

or

re-draw the triangle and set up another proportion

2nd question: want: $\frac{d(x-y)}{dt}$

$$\frac{15}{x} = \frac{6}{x-y}$$

$$15(x-y) = 6x$$

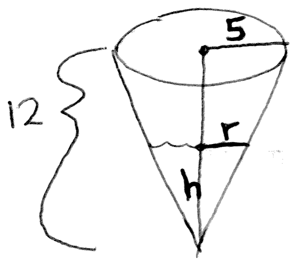
$$15\frac{d(x-y)}{dt} = 6\frac{dx}{dt}$$

$$\frac{d(x-y)}{dt} = \frac{6}{15} \cdot \frac{25}{3}$$

$$\boxed{\frac{d(x-y)}{dt} = \frac{10}{3} \text{ ft/sec}}$$

$\boxed{ex5}$ A conical tank is 10 ft across the top & 12 ft deep. If water is flowing in at a rate of 10 ft³/min find the rate of change of the height of the water when the water is 8 feet deep.

① Draw



given: $\frac{dV}{dt} = 10$

want: $\frac{dh}{dt}$ when $h = 8$

② Relate the sides: use the Volume formula!

$$V = \frac{1}{3}\pi r^2 h$$

③ Implicit differentiation

product rule

$$\frac{dV}{dt} = \frac{1}{3}\pi r^2 \cdot \frac{dh}{dt} + h \cdot \frac{2}{3}\pi r \cdot \frac{dr}{dt}$$

→ Missing r and $\frac{dr}{dt}$

$$\frac{r}{8} = \frac{5}{12}$$

$$r = \frac{10}{3}$$

$$\frac{r}{h} = \frac{5}{12}$$

$$12r = 5h$$

$$12\frac{dr}{dt} = 5\frac{dh}{dt}$$

$$\frac{dr}{dt} = \frac{5}{12}\frac{dh}{dt}$$

④ Plug in

$$10 = \frac{1}{3}\pi\left(\frac{10}{3}\right)^2 \cdot \frac{dh}{dt} + 8 \cdot \frac{2}{3}\pi \cdot \frac{10}{3} \cdot \frac{5}{12}\frac{dh}{dt}$$

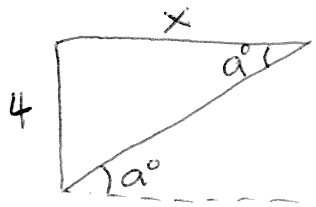
$$10 = \frac{dh}{dt}\left(\frac{100\pi}{27} + \frac{200\pi}{27}\right)$$

Units!

$$\boxed{\frac{dh}{dt} = \frac{9}{10\pi} \text{ ft/min}}$$

$\boxed{\text{ex6}}$ A plane at an altitude of 4 km passes directly over a tracking telescope on the ground. When the angle of elevation is 60°, it is observed that the angle is decreasing by 30°/min. How fast is the plane traveling?

① Draw



given: not much, just 4...

want: $\frac{dx}{dt}$ when $a = 60$; $\frac{da}{dt} = -30$

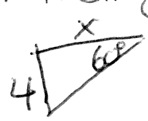
② Set up:

$$\tan a = \frac{4}{x}$$

③ Implicit differentiation

$$\sec^2 a \, \frac{da}{dt} = -4x^{-2} \frac{dx}{dt}$$

→ missing x



$$\tan 60 = \frac{4}{x} \qquad \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{4}{x}$$

$$x = \frac{4}{\sqrt{3}}$$

④ Plug in

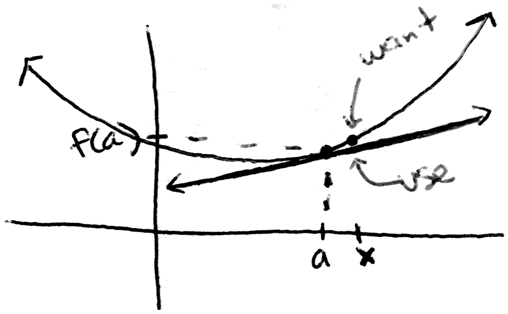
$$\sec^2 60 \cdot -30 = -4\left(\frac{4}{\sqrt{3}}\right)^{-2} \cdot \frac{dx}{dt}$$

$$2^2 \cdot -30 = \frac{-2}{16} \frac{dx}{dt}$$

$$\boxed{\frac{dx}{dt} = 160 \text{ km/min}}$$

VI. Linear Approximation (3G)

Use the tangent line at $(a, f(a))$ as an approximation/estimation of the curve $f(x)$ when x is near a.



equation of the tangent line

linear approximation

ex 1 Find the linearization of the function $f(x) = \sqrt{x+3}$ at $a = 1$ and use it to approximate the numbers $\sqrt{3.98}$ and $\sqrt{4.05}$.

① Find tangent line:

$$f'(x) = \frac{1}{2}(x+3)^{-1/2}$$

$$f'(1) = \frac{1}{2}(1+3)^{-1/2} = \frac{1}{2} \cdot \frac{1}{\sqrt{4}} = \frac{1}{4} = m$$

$$f(1) = \sqrt{1+3} = 2 \qquad (1, 2) \leftarrow \text{point}$$

$$2 = \frac{1}{4}(1) + b$$

$$\frac{7}{4} = b$$

$$y = \frac{1}{4}x + \frac{7}{4}$$

$$\boxed{L(x) = \frac{1}{4}x + \frac{7}{4}}$$

this will approximate the y-values of $f(x) = \sqrt{x+3}$ for x-values close to 1.

② Approximate

$\sqrt{3.98}$; $\sqrt{x+3} = \sqrt{3.98}$

$x = .98$

close to 1

$$L(.98) = \frac{1}{4}(.98) + \frac{7}{4} = \boxed{1.995}$$

$\sqrt{4.05}$; $\sqrt{x+3} = \sqrt{4.05}$

$x = 1.05$

close to 1

$$L(1.05) = \frac{1}{4}(1.05) + \frac{7}{4} = \boxed{2.0125}$$